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What If $\Omega \neq 2q$?

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We discuss various possible scenarios where $\Omega \neq 2q$, where Ω stands for the density parameter and q for the deceleration parameter. We further estimate the corrections necessary when a variable cosmological constant is considered in the theory.

Many textbooks (e.g., Wald, 1984) show that, in general relativity theory, the Friedmann–Robertson–Walker (FRW) cosmology yields, for a pressureless universe,

$$\Omega = 2q \tag{1}$$

where Ω is the density parameter and q is the deceleration parameter, defined by

$$\Omega = \frac{\rho}{\rho_c} = \frac{\rho}{3H^2/8\pi G} \tag{2}$$

$$q = -\frac{RR}{R^2} \tag{3}$$

Here ρ stands for the rest-energy density and R is the scale factor in the FRW metric,

$$ds^2 = dt^2 - \frac{R^2(t)}{(1+kr^2/4)^2}\, d\sigma^2 \tag{4}$$

It has to be clearly understood that relation (1) follows from Einstein's field equations only in the absence of a cosmological term Λ, because if $\Lambda \neq 0$ we would have instead (Narlikar, 1983)

$$\Omega = 2q + 2/3\ \Lambda/H^2 \tag{5}$$

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It is usually accepted that H, Hubble's parameter $\dot{R}/R$, is a function of time, and, in general,

$$H \cong \frac{1}{(1+q)t} \tag{6}$$

This is an exact relation when $q = \text{const}$, and is an approximate result valid in the same way for a certain lapse of time where we can have an approximately constant deceleration parameter (Berman, 1983; Berman and Gomide, 1988). For the present universe, where the inflationary model (Guth, 1981) seems to point to $\Omega = 1 = \text{const}$ for a large span of time, we would expect that all the terms in (5) are constant, and this points to a time-varying Λ,

$$\Lambda = At^{-2} \qquad (A = \text{const}) \tag{7}$$

so that

$$\Omega = 2q + 2/3\ A(1+q)^2 \tag{8}$$

Relation (7) has also recently been found in a number of different scenarios (Berman *et al.*, 1989; Berman, 1990*a*,*b*, 1992*a*; Berman and Som, 1990; Bertolami, 1986; Ng, 1991); on the other hand, if we find that

$$\Omega \neq 2q \tag{9}$$

we cannot rule out other possibilities, such as that the universe is a Brans-Dicke one (Berman, 1992*b*) or an Einstein-Cartan one (Berman, 1992*c*), in which cases we would have to add to the right-hand side of equation (5) terms like $128\pi^2 S^2/3H^2$, where S stands for the spin magnitude $S^2 \equiv S_{ik}S^{ik}$ (where S^{ik} is the spin tensor), or like $(2\omega/3H^2)(\dot{G}/G)^2$, where ω is the Brans-Dicke coupling constant and G stands for Newton's gravitational "constant."

Now, let us return to relations (7) and (8), and find a limit to the maximum possible present value of the constant A.

Taking (Barrow, 1990)

$$\Lambda_0 \lesssim 10^{-120} l_{Pl}^{-2} \tag{10}$$

where $l_{Pl}^2 \cong 10^{-66}\ \text{cm}^2$, we find

$$\Lambda_0 \lesssim 10^{-54}\ \text{cm}^{-2} \tag{11}$$

This means, considering

$$t_0 \approx H_0^{-1} \approx 2/3 \times 10^{17}\ \text{sec}$$

that

$$A_{\max} \approx \Lambda_0 t_0^2 = 4/9 \times 10^{-20}\ \text{sec}^2\ \text{cm}^{-2}$$

Relation (8), however, is written in relativistic units, so that we have to multiply A by the square of the speed of light in vacuum, and we have, in nonrelativistic units,

$$A_{nr} \lesssim c^2 A \approx 4/9 \times 10^{-20} \times (3 \times 10^{10})^2 \approx 4$$

Then we see that

$$\Omega \lesssim 2q + \alpha(1+g)^2 \qquad (12)$$

In (12) we have discarded the factor 4, and substituted a constant α which is of the order unity. We have shown frameworks where (9) is true, thus answering the question in the title to this paper.

From the fact that

$$\Omega \geq 0 \qquad (13)$$

because of the positivity of energy condition

$$\rho \geq 0 \qquad (14)$$

we can estimate α. We have

$$\alpha \gtrsim -\frac{2q}{(1+q)^2} \qquad (15)$$

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